Abstractions for Dynamics of Automata Networks (brief overview)

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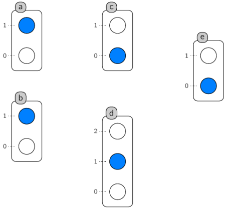
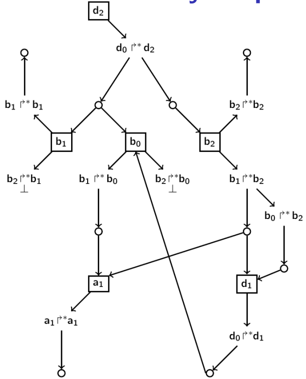
Overview

Automata network



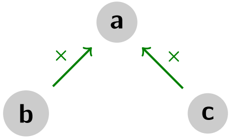
Local Causality Graph



Over-approximation of reachability
Under-approximation of reachability
Under-approximation of cut sets

Goal-oriented model reduction

Transition-centered specification



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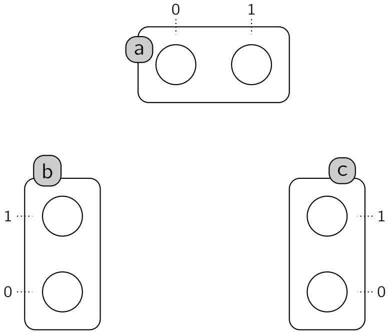

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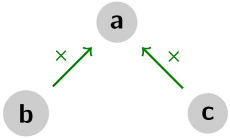

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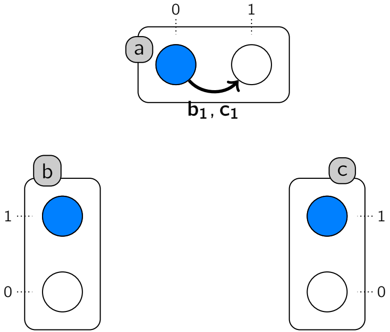

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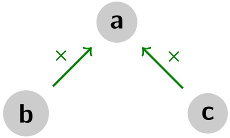

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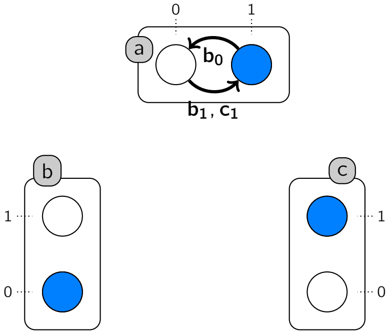

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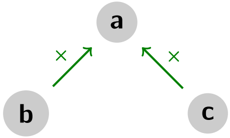

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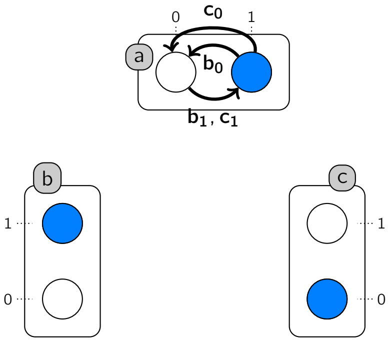

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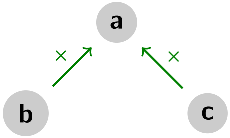

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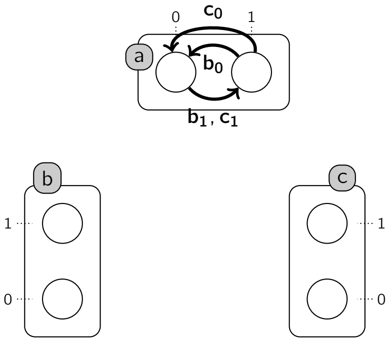

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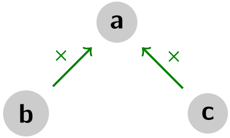

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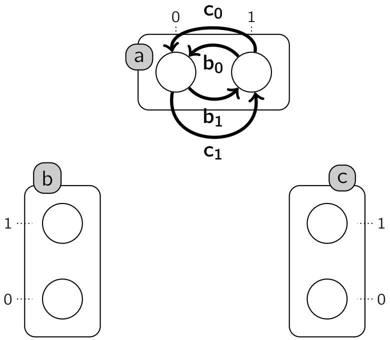

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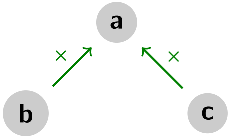

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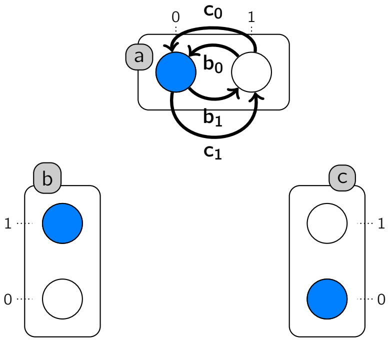

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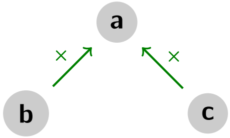

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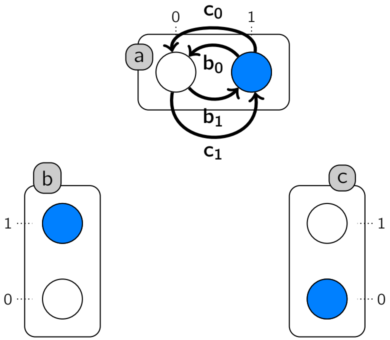

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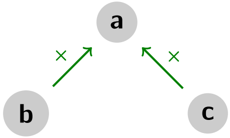

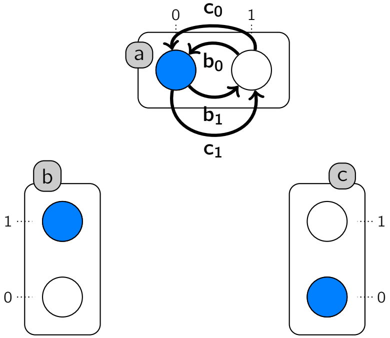

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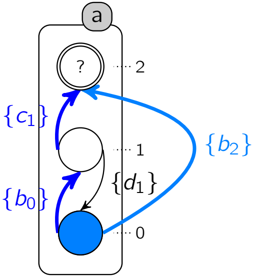
Local Causality



$$\text{local-cause}(a_0 \rightarrow^* a_2) = \{a_0 \xrightarrow{b_0} a_1 \xrightarrow{c_1} a_2,\ a_0 \xrightarrow{b_2} a_2\}$$

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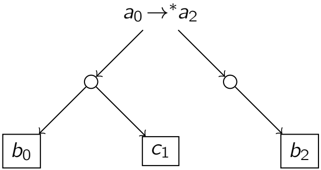

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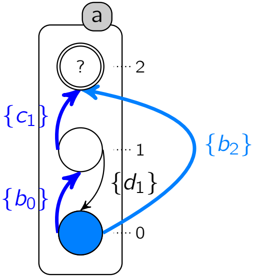

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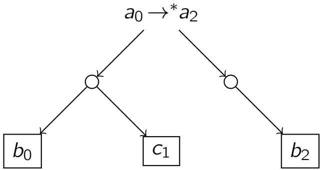
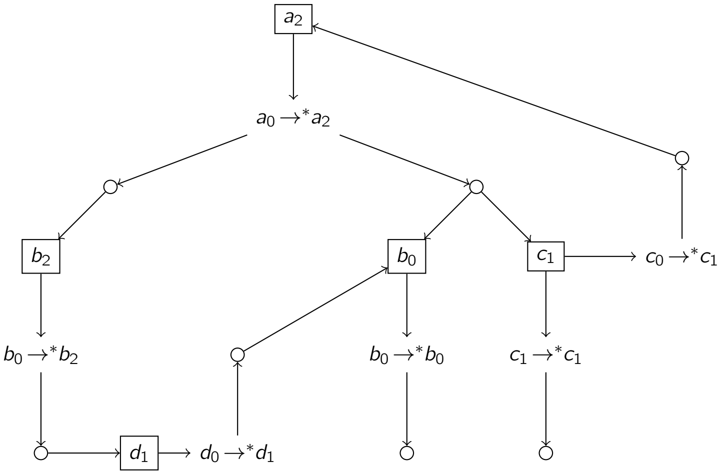


For any trace π starting at some global state s with $a_0 \in s$ and reaching a_2:

- either $a_0 \xrightarrow{b_0} a_1 \xrightarrow{c_1} a_2$ or $a_0 \xrightarrow{b_2} a_2$ is a sub-trace of π;
- either b_1 and c_0, or b_2 are reached before a_2 in π.

Local Causality Graph

- Causality of a_2.
- Initial context $\varsigma = \{a \mapsto \{0\}; b \mapsto \{0\}; c \mapsto \{0,1\}; d \mapsto \{0\}\}$.



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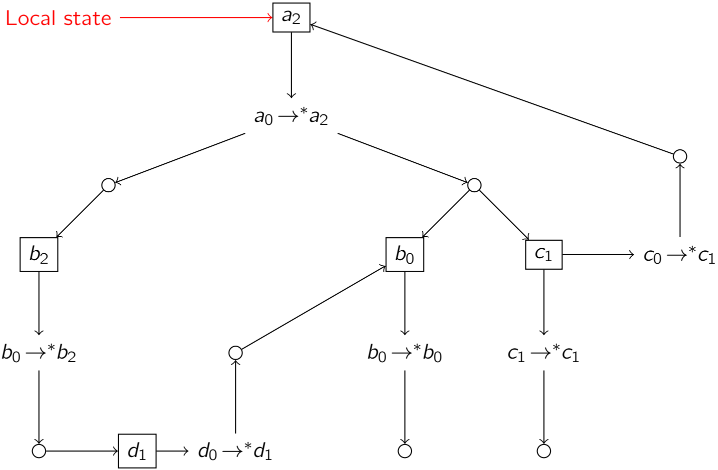

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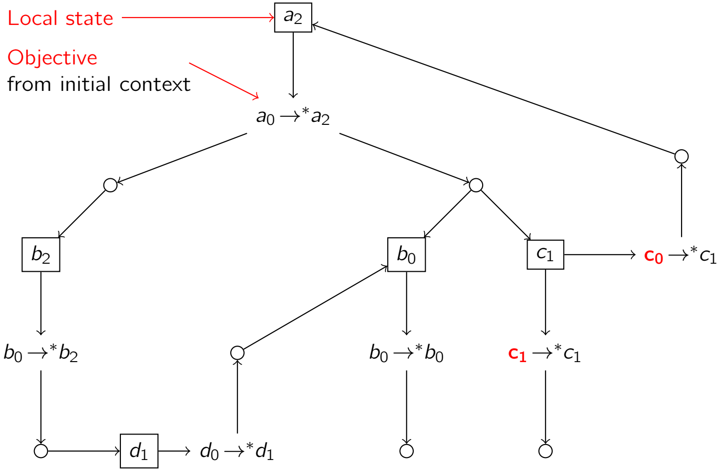

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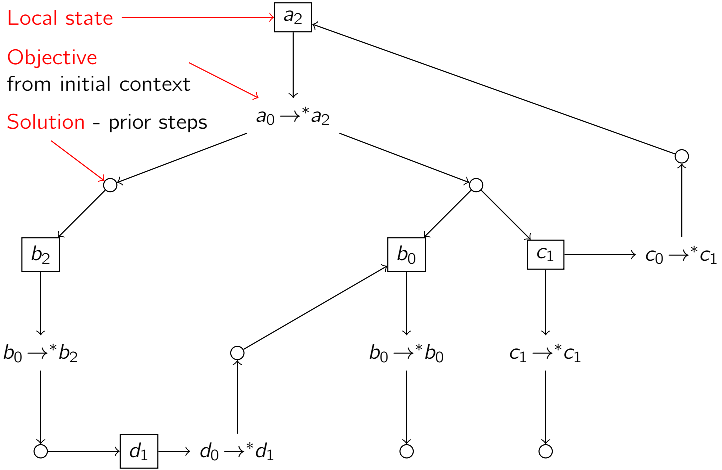

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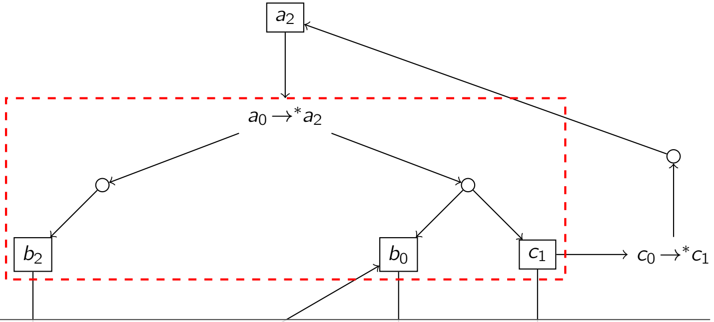


Necessary condition
The occurrence of a_2 from a_0 necessarily requires the prior occurrence of b_2 *or* of b_0 and c_1.

Results using local causality analysis

Verification for transient reachability

- Necessary or sufficient conditions;
- Low complexity (polynomial with nb automata, exp with automaton size);

Cut-sets for transition reachability (towards control)

- Identify sets of mutations to break a transition reachability;
- Under-approximation of cut-sets
- Applied to models with 10,000 nodes.

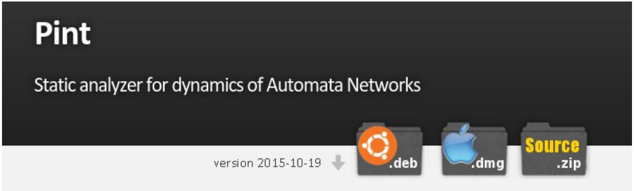
Goal-oriented model reduction

- Given a reachability property (goal), identify **"useless" transitions**.
- Preserves **all minimal traces** for the goal reachability.
- Low complexity (polynomial with nb automata, exp with automaton size);

⇒ pre-processing before model-checking
⇒ benchmarks show effective model reduction
(research report: `https://hal.archives-ouvertes.fr/hal-01149118`)

Software: Pint

http://loicpauleve.name/pint



- Input: automata networks
  - implemented in `LogicalModels` (shipped with `GINsim`)
- Command line tools:
  - Static analysis for reachability
  - "Cut sets" for reachability
  - Model reduction w.r.t. reachability property
  - Fixed points
  - Inference of Thomas parameters
  - Interface with model-checkers (NuSMV, ITS, mole).
- OCaml library (C/C++ bindings in progress)

(secret project: GUI)

Conclusion/Perspectives

Static analysis of automata network dynamics

- Local **causality analysis**, summarized in a compact graph
- Reasonning on **necessary/sufficient conditions** for transitions
- Multiple applications for **transient reachability** analysis
- Can address very large networks (several 1,000 nodes).

Goal-oriented model reduction

- Embed in model-checking approaches (Petri net unfoldings, etc.)
- Embed in **model identification** (static pruning of model space).

Cell reprogramming

- Large models
- Boolean abstraction well suited (experimental validations)
- Causality analysis for **deriving potential reprogramming determinants**?

Thanks for your attention!