Boolean Networks:
Beyond Generalized Asynchronicity

Thomas Chatain[1], Stefan Haar[1], *Loïc Paulevé*[2]

[1] LSV, ENS Paris-Saclay, Inria Saclay, France

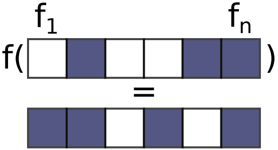
[2] CNRS, LRI, Univ Paris-Sud, Univ Paris-Saclay, France

AUTOMATA 2018, Gent, Belgium

Introduction

Boolean Network (BN) $f : \mathbb{B}^n \to \mathbb{B}^n$

Configuration: $x \in \mathbb{B}^n$



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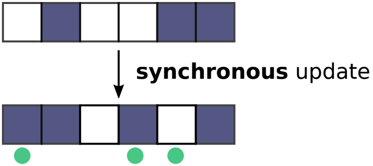

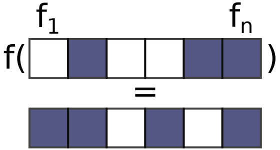

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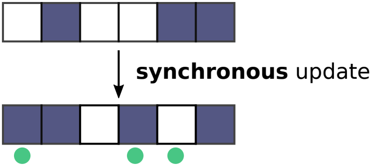
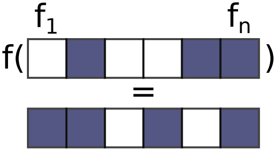
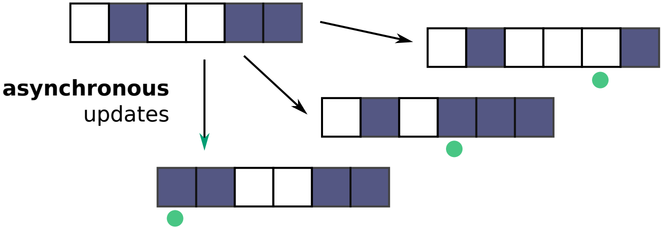
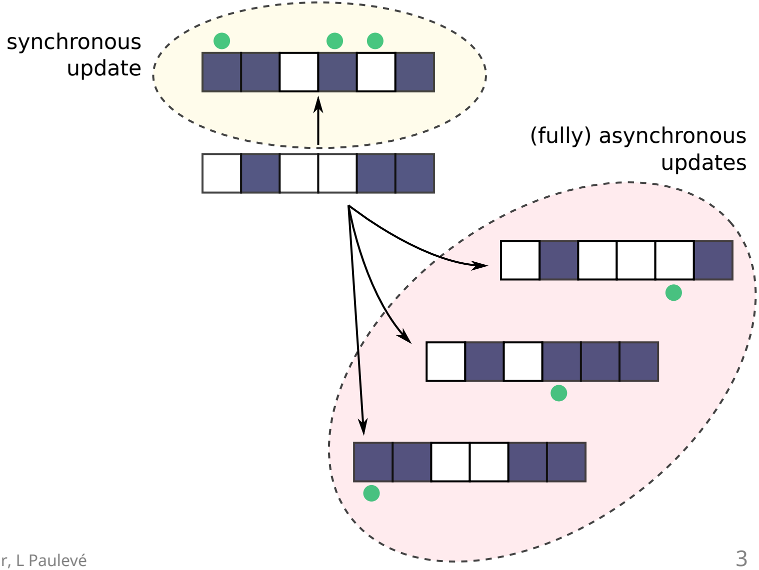






Generalized Asynchronicity



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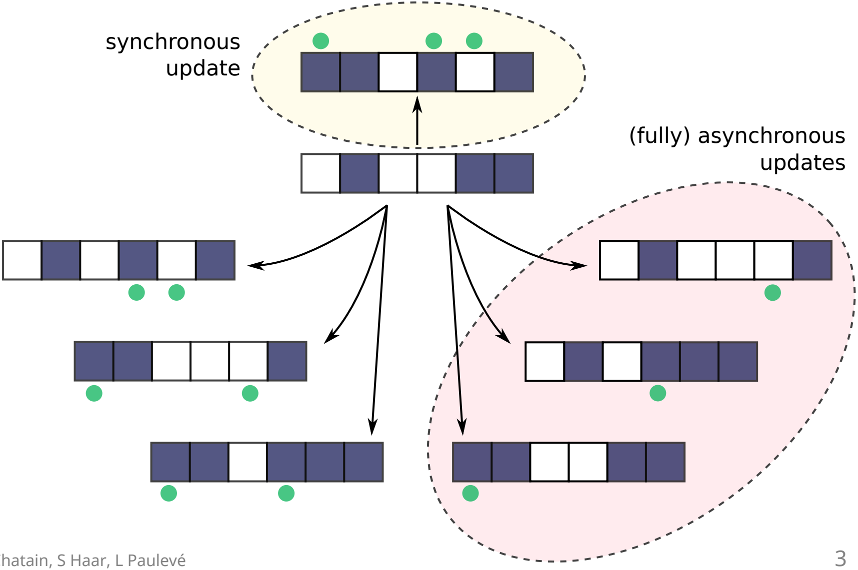
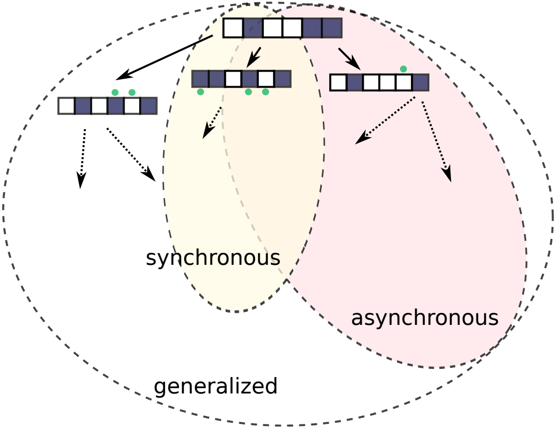


Reachable configurations



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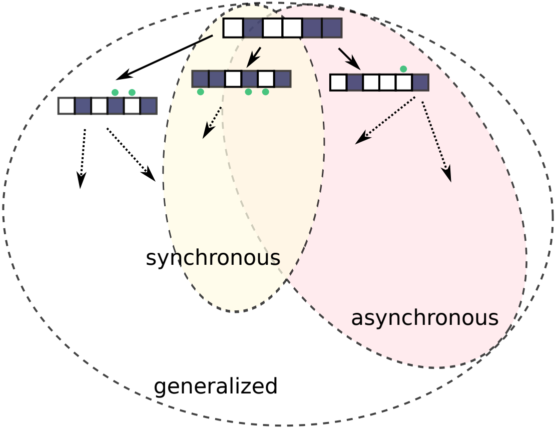


Can we reach configurations
beyond generalized asynchronicity?

Contribution

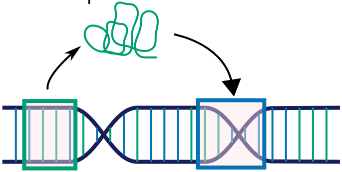
Can we reach configurations
beyond generalized asynchronicity?

- inspiration from Petri nets/concurrency theory
- BN encoding of "interval semantics"
  $\Rightarrow$ allow delay of update application
  $\Rightarrow$ enable new reachable configurations
- impact for modelling biological networks
  $\Rightarrow$ towards consistent Boolean abstractions
  w.r.t. refined dynamics

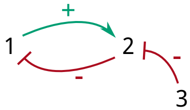
Boolean networks as **abstractions** of dynamics of **biological networks**

Boolean networks for biological processes

Example with gene regulation



Influence graph



Boolean networks for biological processes

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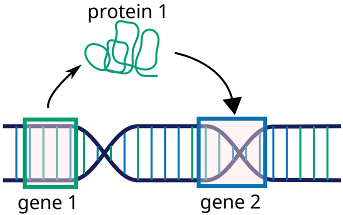

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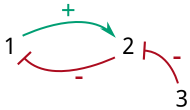

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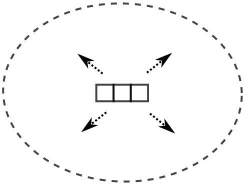


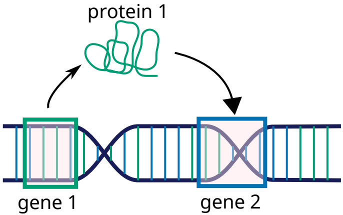
Boolean network

$$f_1(x) \triangleq \cdots$$
$$f_2(x) \triangleq \cdots$$
$$f_3(x) \triangleq \cdots$$

+ update mode =

Boolean networks for biological processes

Example with gene regulation



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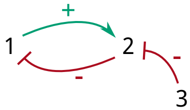

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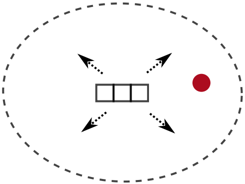

Validation w.r.t. observations (e.g. time series data)

⇒ we expect measurements match with reachable configurations

Motivating example
(embedded in many actual biological networks)

$$f_1(x) \triangleq \neg x_2$$

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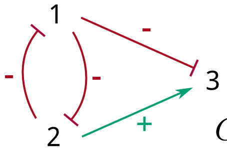
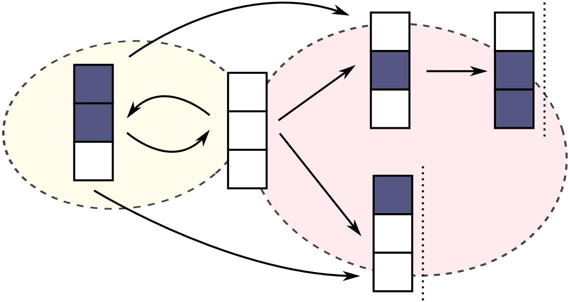
$$f_3(x) \triangleq \neg x_1 \wedge x_2$$



$G(f)$



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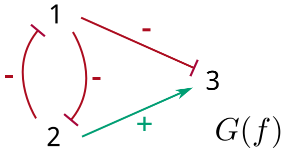
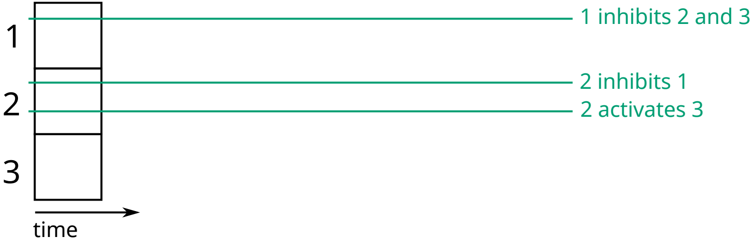


Compatible continuous/multilevel dynamics:



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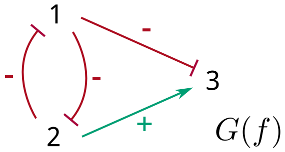

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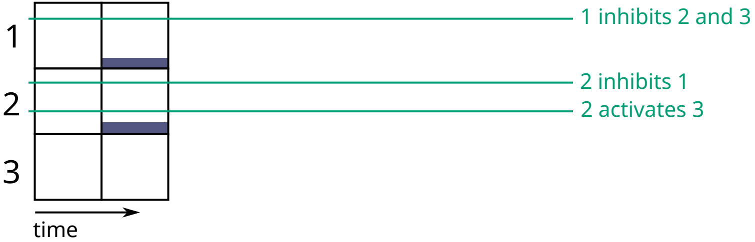

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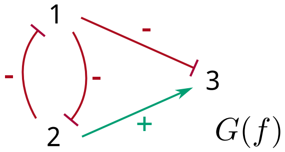

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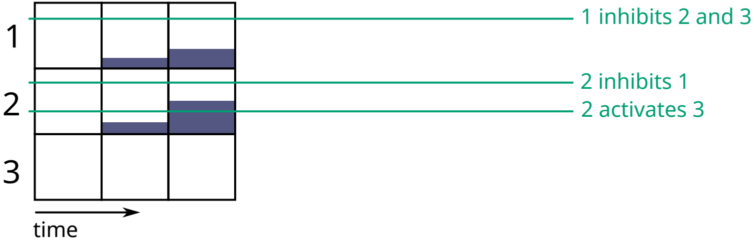

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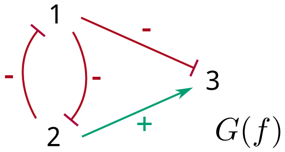

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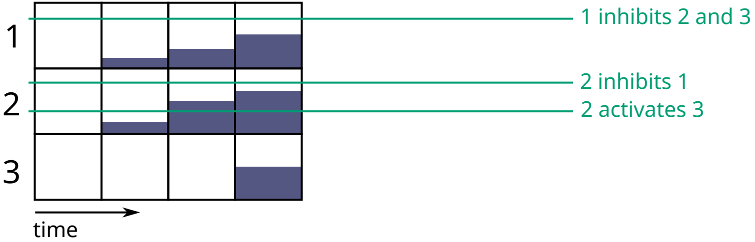

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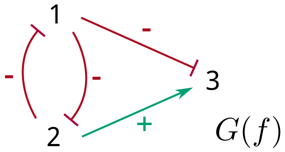

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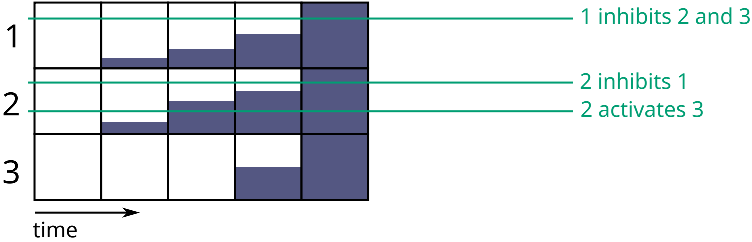

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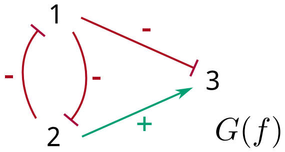

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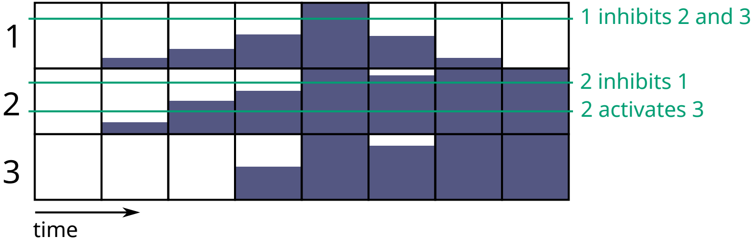

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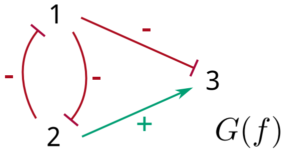

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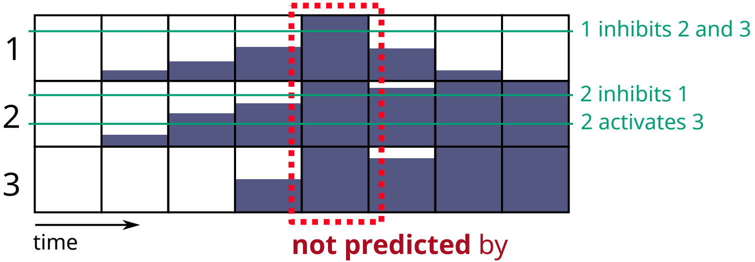


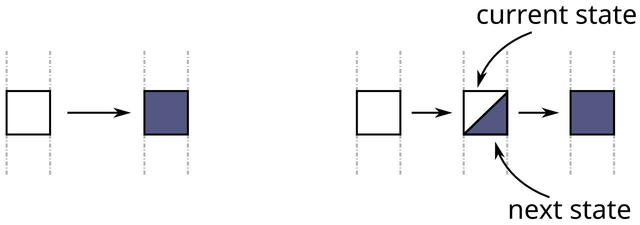
not predicted by
generalized asynchronicity!

Interval Semantics
for Boolean networks
enabling new behaviours

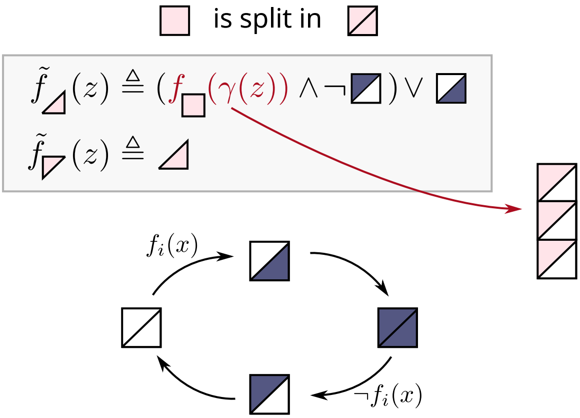
Interval semantics

- introduced for Petri nets [Chatain et al., Petri Nets'15]
- delay between firing and application of state change

  ⇒ allow interleaving other state changes
- once fired, a state change is committed to happen

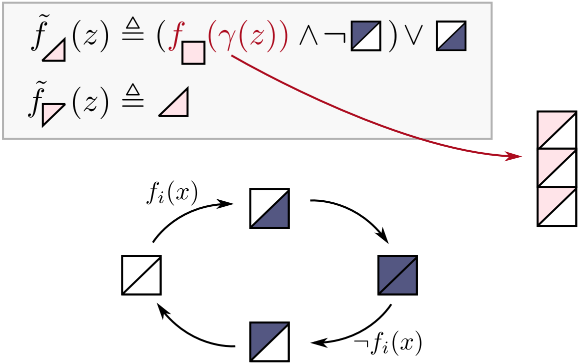


Interval semantics with Boolean networks



Interval semantics with Boolean networks

is split in



weakly simulates generalized asynchronous updating:

$$\forall x, y \in \mathbb{B}^n, \quad x \xrightarrow[gen]{f} y \Longrightarrow \tilde{x} \xrightarrow[async]{\tilde{f}}{}^{*} \tilde{y}$$

Application to motivating example

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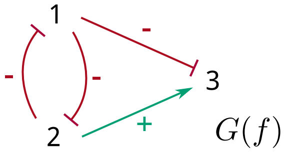

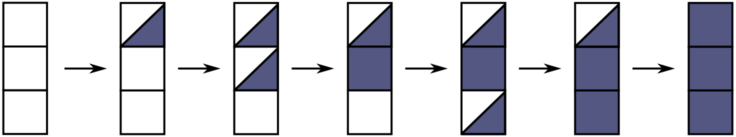

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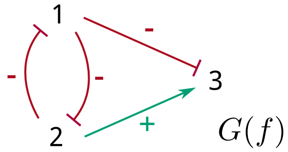
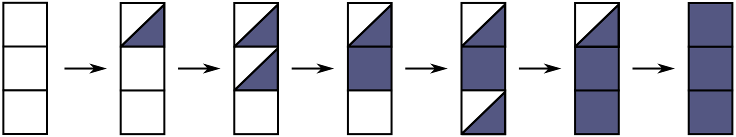


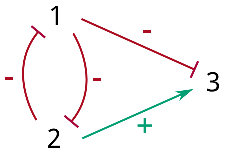


⇒ valid with respect to multivalued refinement

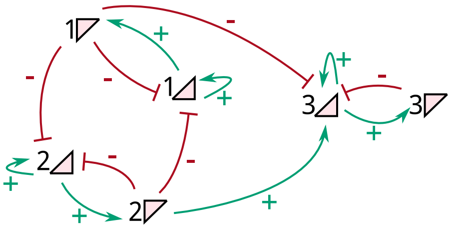
Properties of the encoding

- weakly simulates generalized asynchronous updating
- equivalence of fixpoints
- preserve major features of the influence graph
  (from which can be derived dynamical properties)

$G(f)$



$G(\tilde{f})$



Conclusion

- Encoding of Interval Semantics (Petri nets) with fully asynchronous Boolean networks
- Subsumes generalized asynchronous updates
- Enable new reachable configurations, while preserving architecture of causal relationships

⇒ impact for validating BN models in systems biology

Beyond generalized asynchronocity and interval semantics?

- Extension which meet with a correct Boolean abstraction of any multivalued refinement [submitted]