Abstractions for Dynamics of Automata Networks

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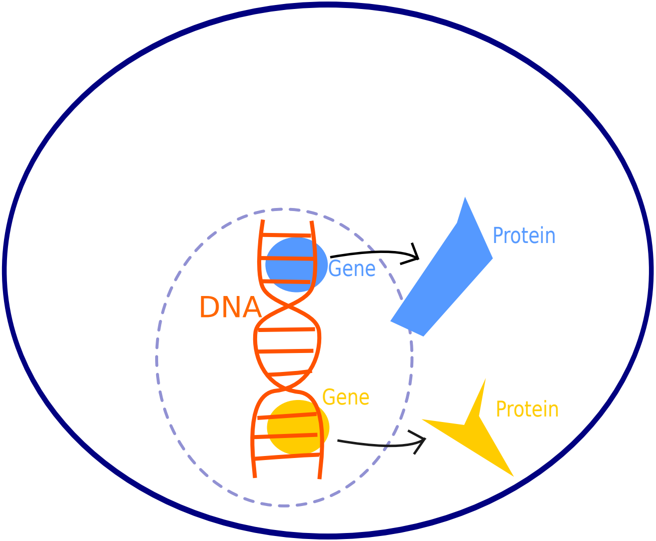
March 30, 2015 - LIFO, Orléans

Biological Interaction Networks



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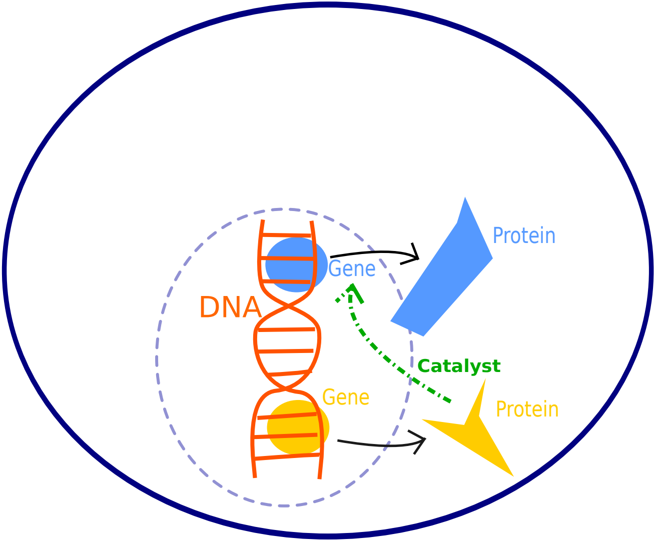

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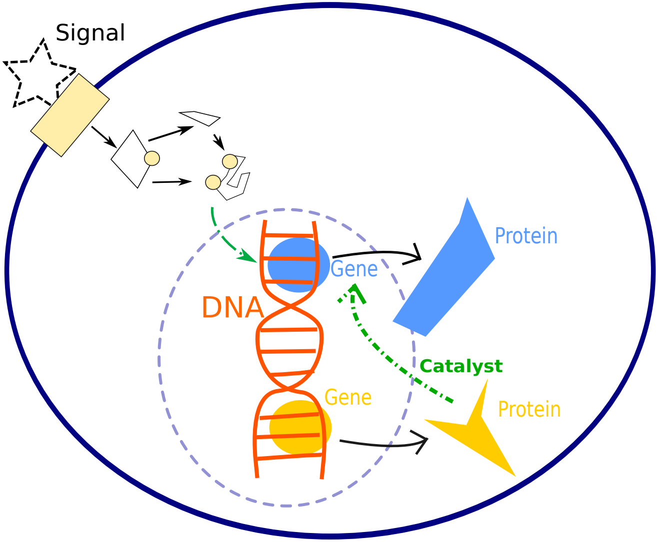
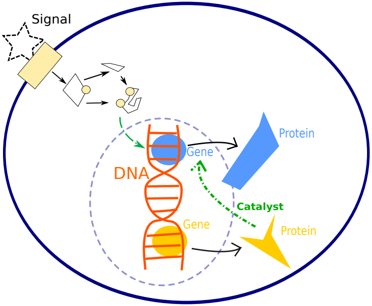


Biological Motivations



Prediction

- Cell response w.r.t. signal+environment
- Long-term behaviours (differentiation)

Control

- Mutations for modifying cell response
- Re-differentiation

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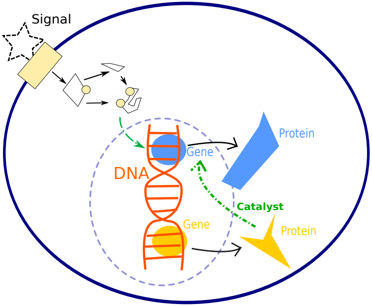

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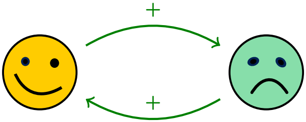
$\Rightarrow$ {
Models of the system **dynamics**
−Formal verification
−Synthesis

Interaction Networks

Qualitative approach

- Interacting Entities having simplistic behaviour (e.g.: happy/sad)
- Positive or negative influences
- Dynamical system: state of entities evolve with time
- Complex system: locally simple, emerging properties hard to predict



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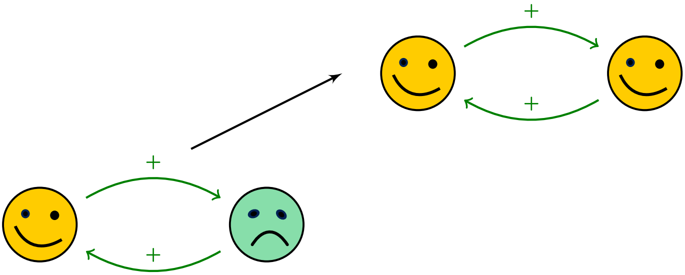

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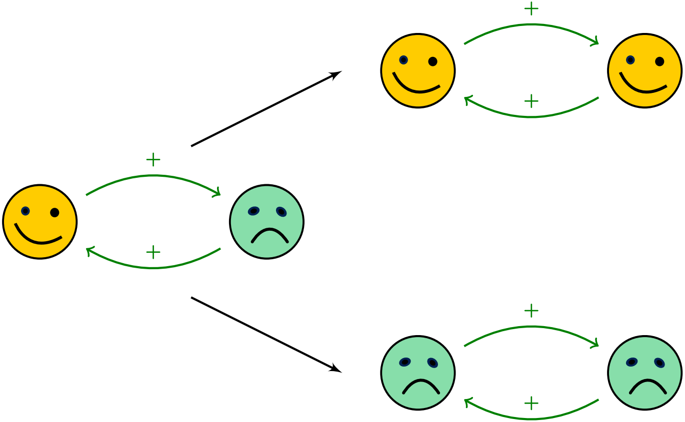

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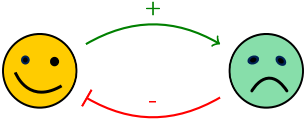

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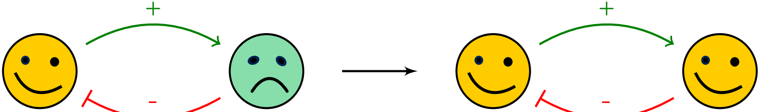

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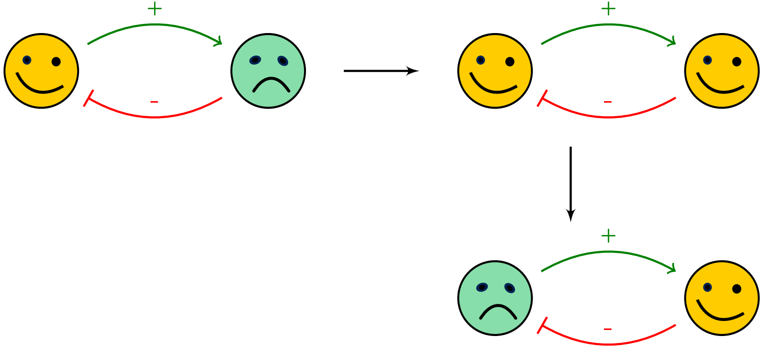

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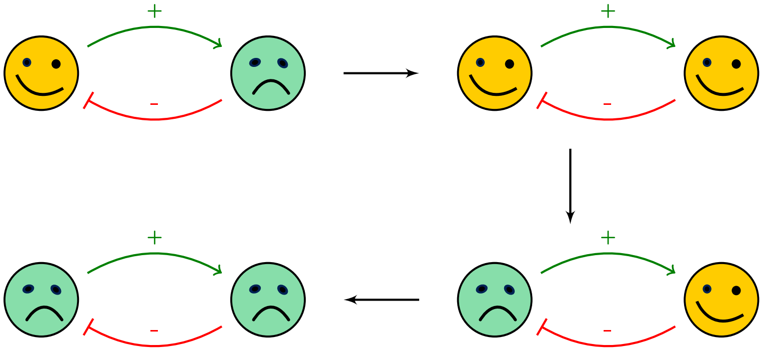

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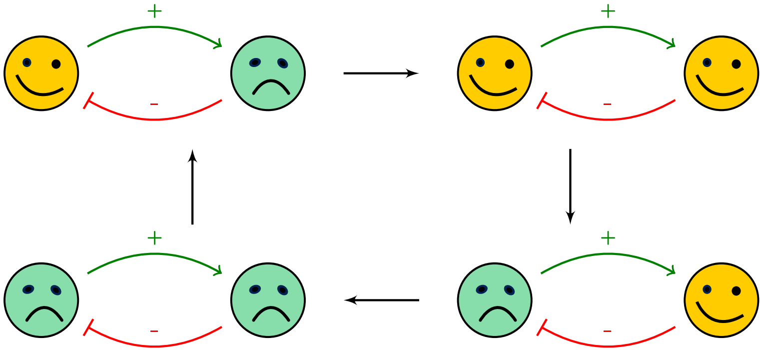
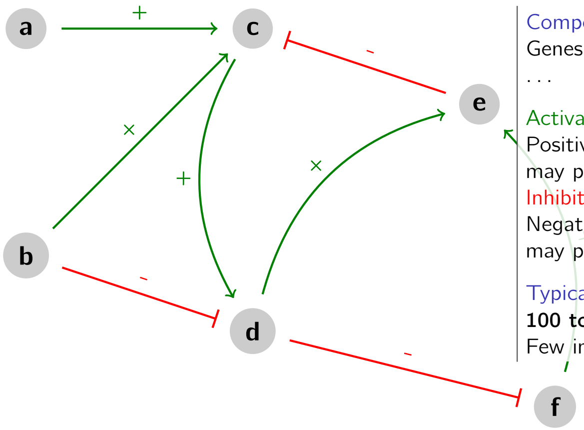


Interaction Networks

E.g., Signalling Networks, Gene Regulatory Networks



Components

Genes, proteins, complexes, . . .

Activations

Positive influence (*a* increase may promote *c* increase).

Inhibitions

Negative influence (*b* increase may promote *d* decrease).

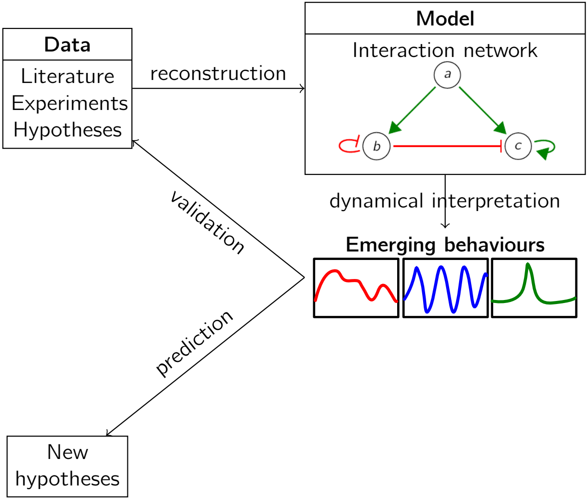
Typical settings

100 to +10,000 components

Few information on kinetics

Formal Methods for Systems Biology

Aim: **understand, analyse, control** emerging dynamics.



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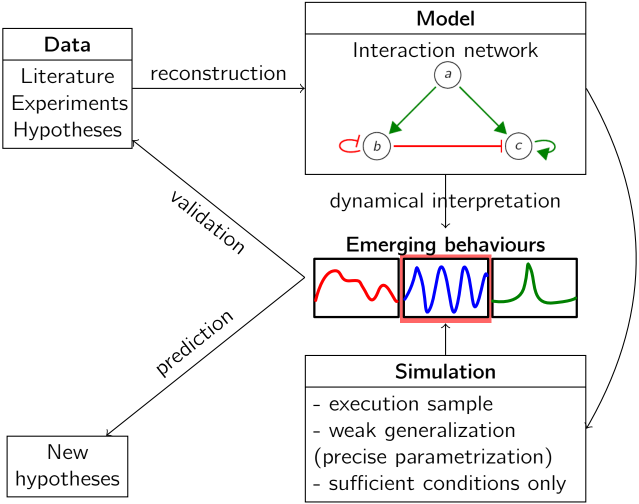

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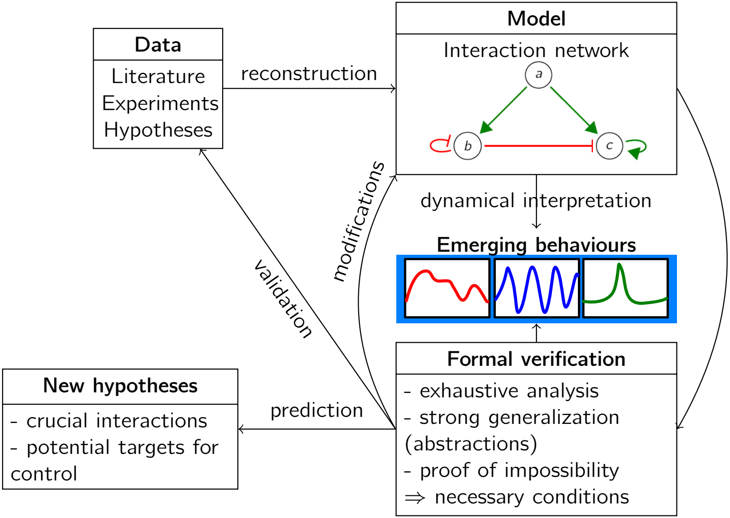
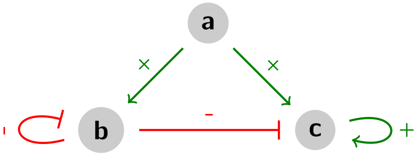


Dynamics of Qualitative Networks

Example in Boolean case



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$\langle a, b, c \rangle$ $\langle 1, 0, 0 \rangle$

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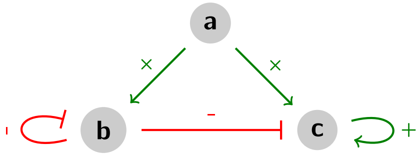

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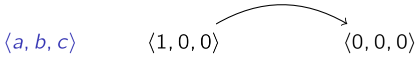

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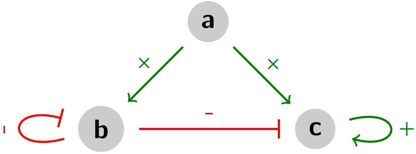

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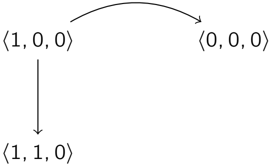

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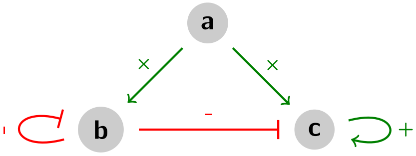

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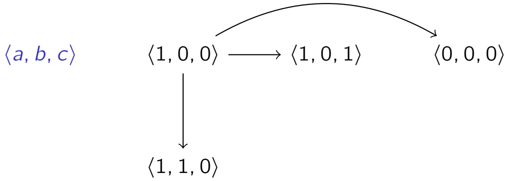

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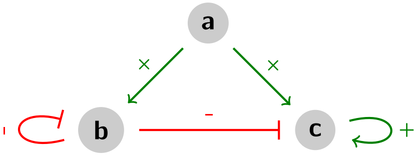

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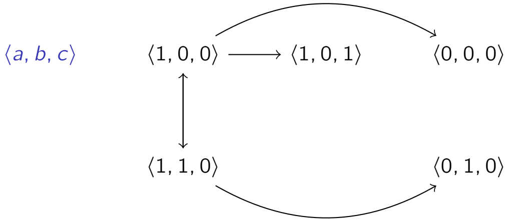

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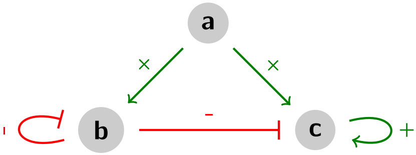

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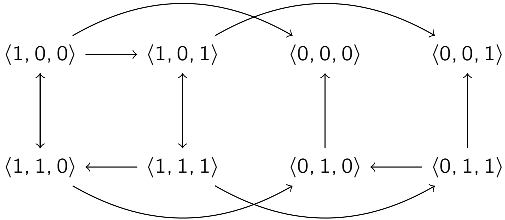

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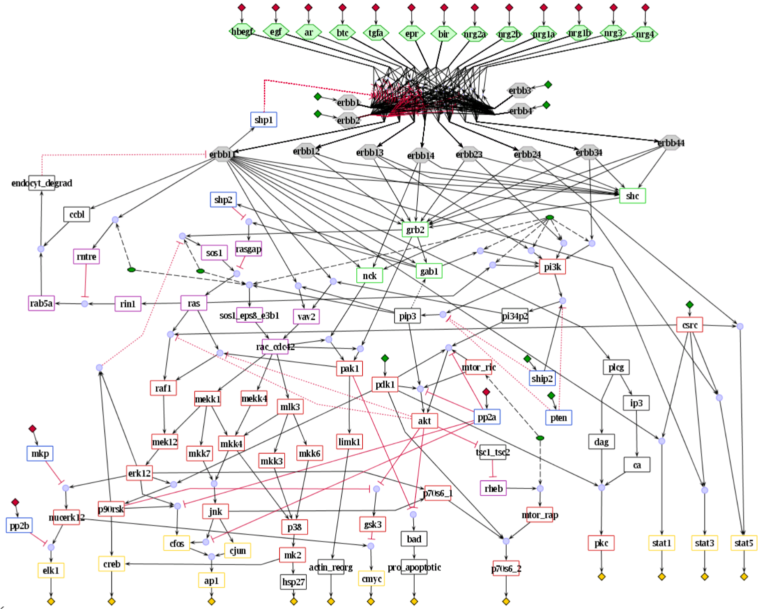
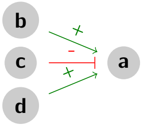
Large Interaction Networks



Issues with Large Interaction Networks

Modelling issues

- Partially-specified interactions.
- Boolean networks need to be fully specified (deterministic Boolean function f_a).
- Intractable enumeration of all models.



Analysis issues

- Combinatorial explosion of behaviours (e.g. 2^{100} - 10^{30} to $2^{10\,000}$ - 10^{3000} states).
- Large range of initial conditions to consider.
- Difficult to extract comprehensive proofs of (im)possibility.

Contributions

Scalable analysis of **transient dynamics** of automata networks

Static analysis

Model $\longrightarrow$ Abstraction $\longrightarrow$ Decision (possibly incomplete)

Key ingredients

- Concurrent systems
- Transition-centered specification
- Causality analysis and abstraction

Results

- Prevent raw model-checking (PSPACE-complete)
- Derive necessary or sufficient conditions from abstractions
- Allow coupling with exhaustive analysis

Outline

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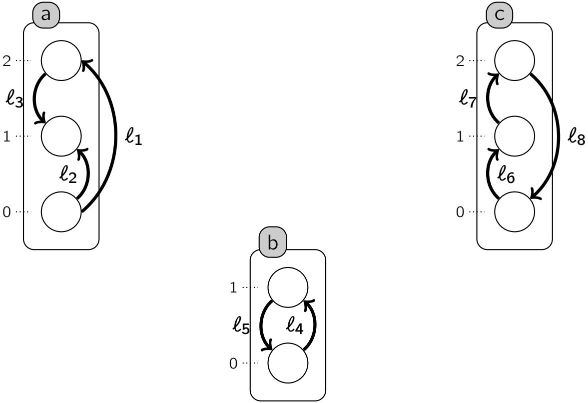
Asynchronous Finite Automata Networks



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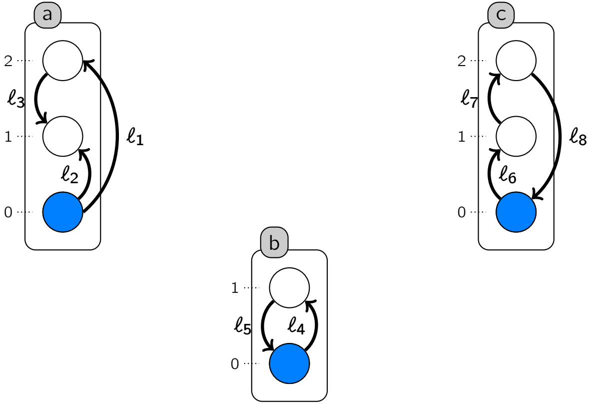

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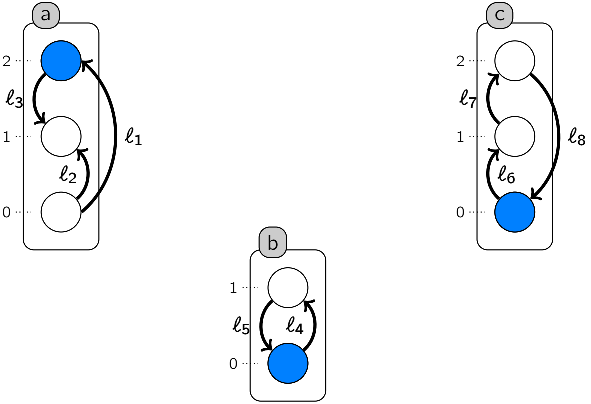

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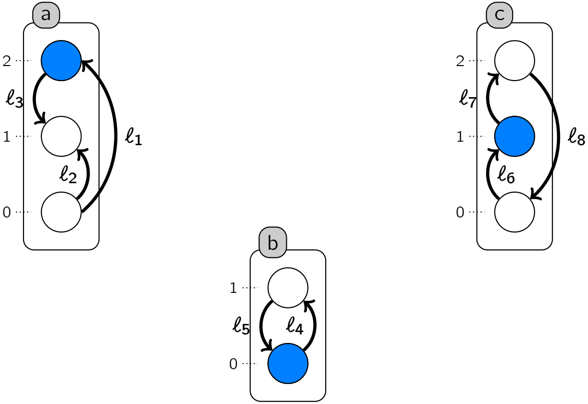

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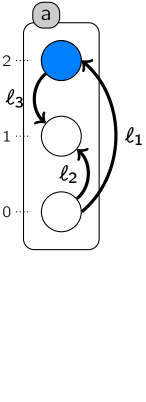

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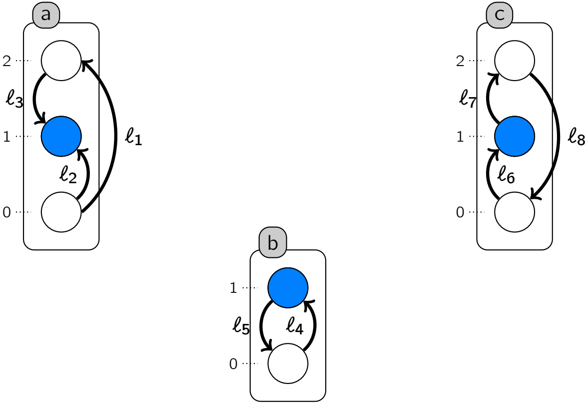

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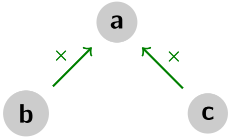

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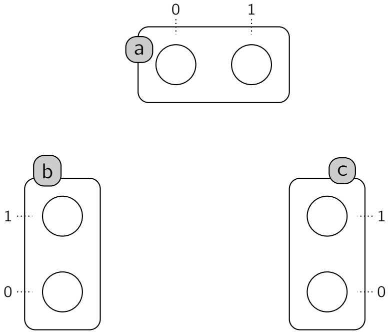

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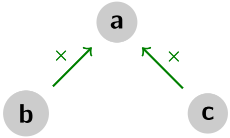

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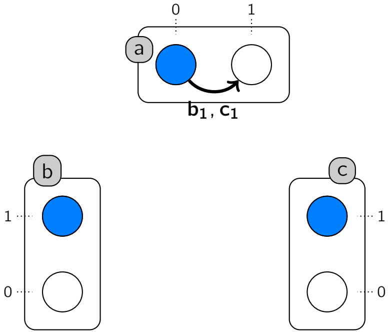

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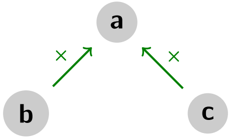

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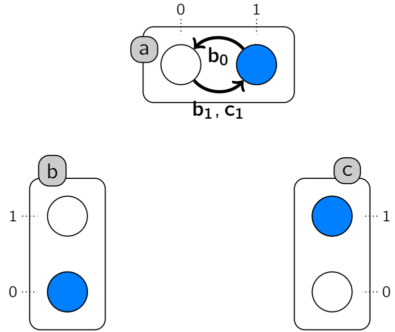

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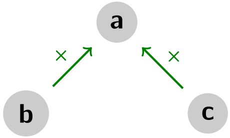

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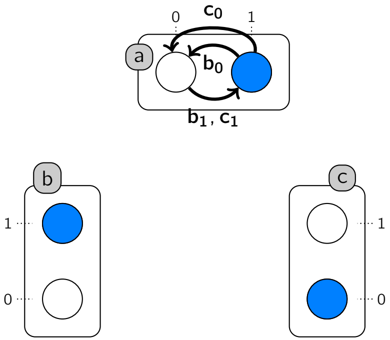

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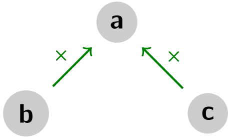

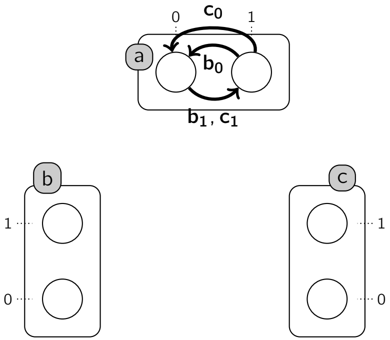

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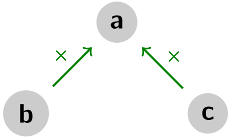

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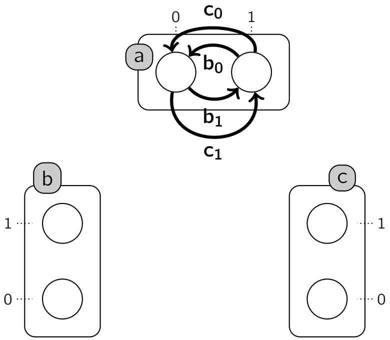
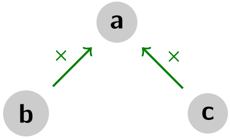


Transition-centered specification



1. $f^a(x) = x[b] \wedge x[c]$

transitions:

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2. Non-deterministic f^a

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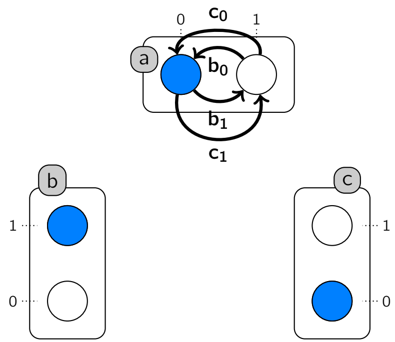

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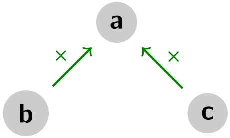

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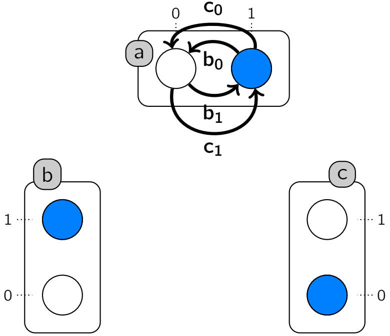

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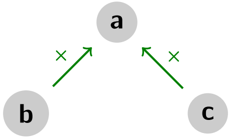

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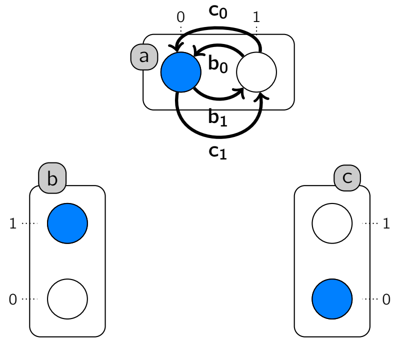
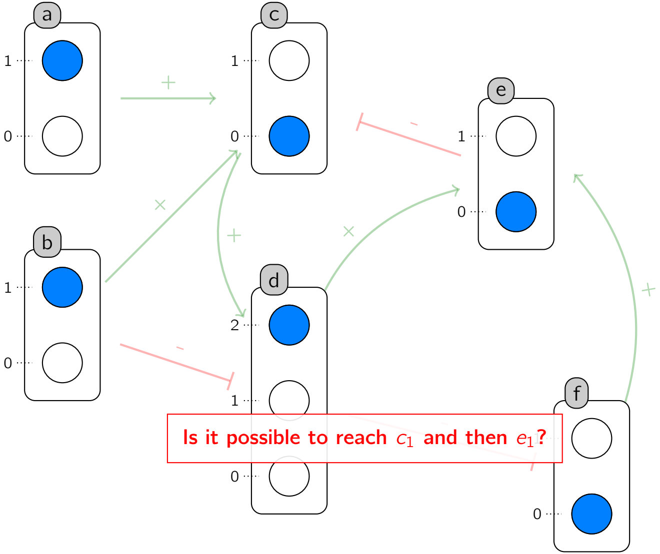


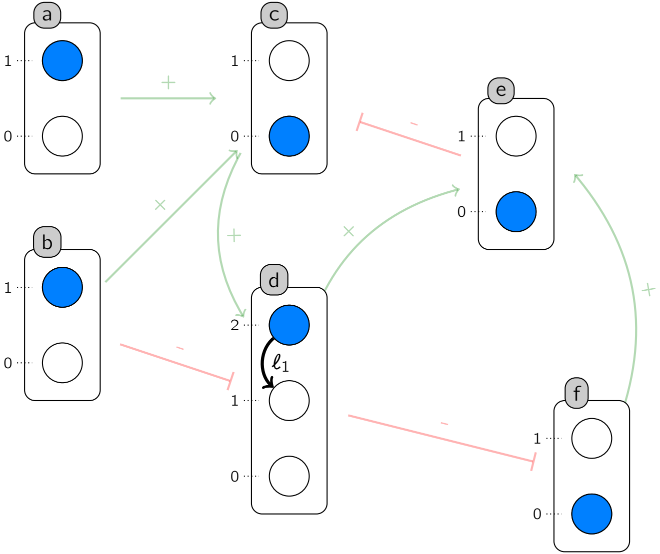
Outline

Reachability

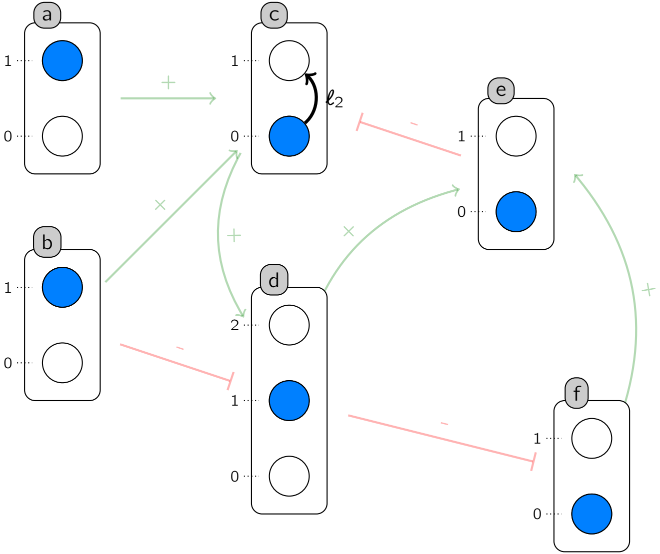


Is it possible to reach c_1 and then e_1?

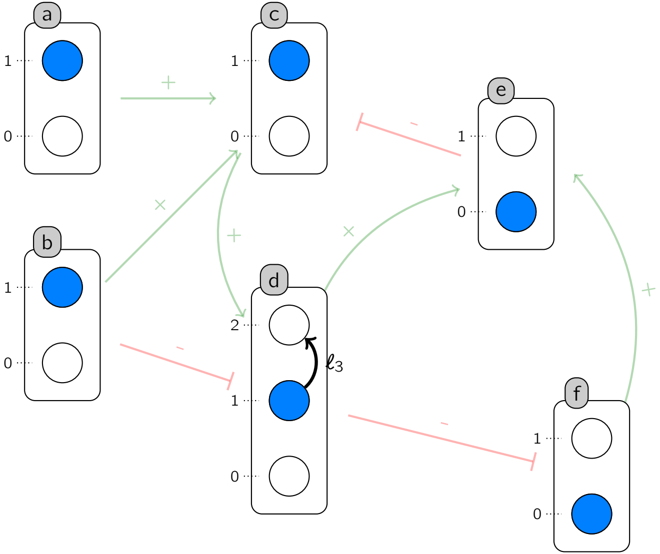
Reachability



Reachability



Reachability



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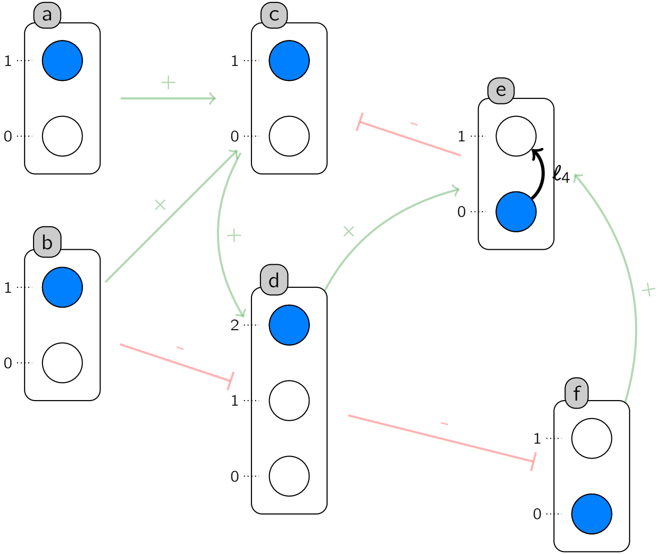

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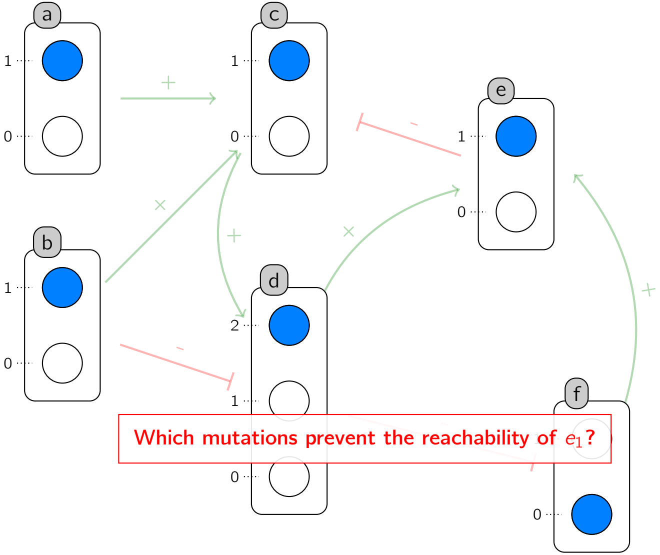
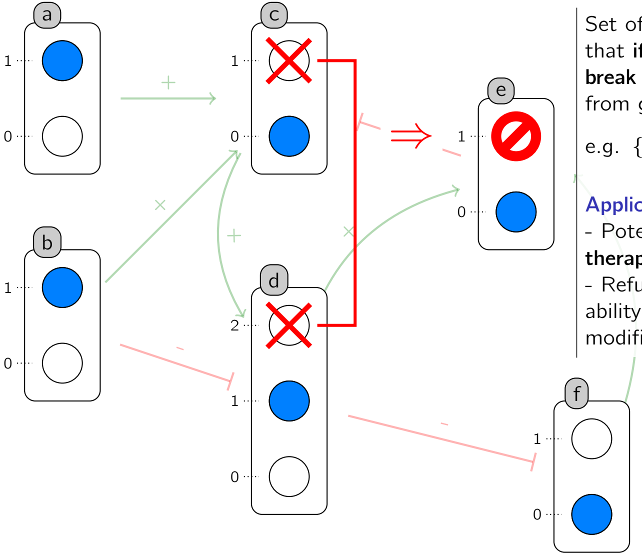


Which mutations prevent the reachability of e_1?

Cut Sets for Reachability



Set of **local states** that **if all disabled break reachability** from given initial states

e.g. $\{c_1, d_2\}$

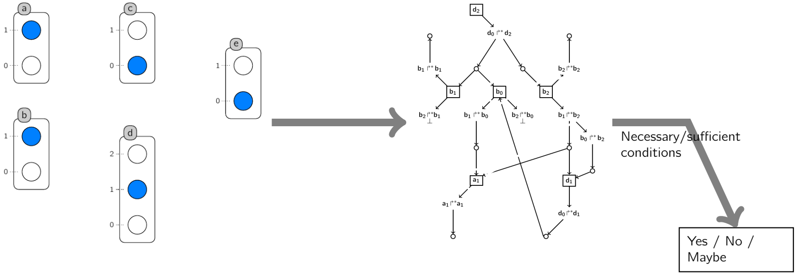
Applications

- Potential **therapeutic targets**
- Refute model if reachability still occurs in the modified (real) system

Reachability Analysis with Abstract Interpretation

Automata Network
+ reachability prop.

Graph of Local Causality



- Over- and under-approximations of local rechability properties.
- Low complexity: poly(nb. automata) $\times$ exp(nb of procs in one automaton)

  $\Longrightarrow$ efficient with a small number of processes per automaton, while a very large number of automata can be handled.

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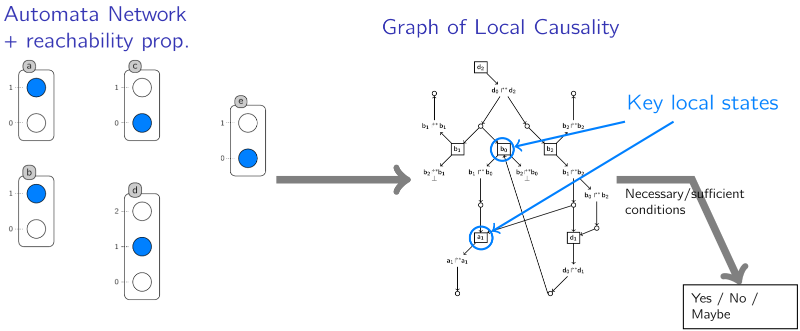


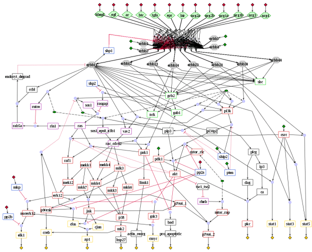
- Over- and under-approximations of local rechability properties.
- Low complexity: poly(nb. automata) × exp(nb of procs in one automaton)
  $\Longrightarrow$ efficient with a small number of processes per automaton, while a very large number of automata can be handled.
- Key local states: necessary for reachability satisfiability (control).

Applications

Large signalling networks - reachability

Model	NuSMV	ITS	**PINT**
EGFR (20)	[3s-KO]	[1s-150s]	**0.007s**
TCR (40)	[1s-KO]	[0.6s-KO]	**0.004s**
TCR (94)	KO	KO	**0.030s**
EGFR (104)	KO	[9mn-KO]	**0.050s**

- Range over initial states / reachability prop.
- In those cases: always conclusive.



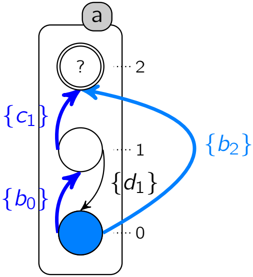
TGF-β signalling - ANR BioTempo, N. Theret (INSERM), G. Andrieux (IRISA)

- 9,000 interaction components
- Identification of key processes for a particular activation
- Dynamics: 2^{9000} states, PINT: reachability $< 1s$, key components $< 10min$

$\Rightarrow$ first formal analysis of dynamics at such a large scale

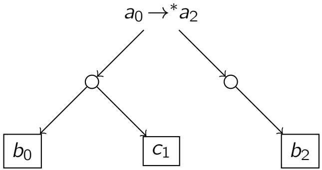
Outline

Local Causality



$$\text{local-cause}(a_0 \rightarrow^* a_2) = \{a_0 \xrightarrow{b_0} a_1 \xrightarrow{c_1} a_2,\; a_0 \xrightarrow{b_2} a_2\}$$

$$\text{local-cause}^{\#}(a_0 \rightarrow^* a_2) = \{\{b_0, c_1\}, \{b_2\}\}$$



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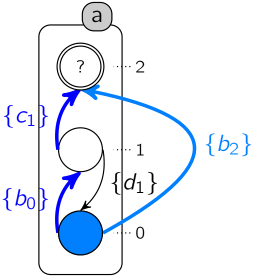

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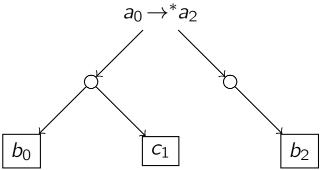
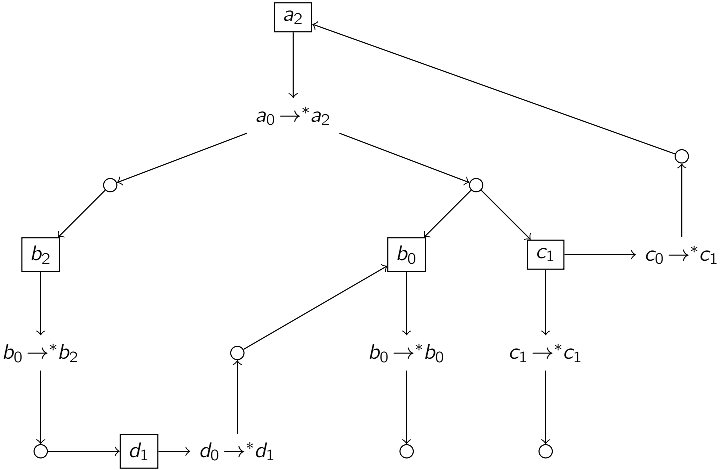


For any trace π starting at some global state s with $a_0 \in s$ and reaching a_2:

- either $a_0 \xrightarrow{b_0} a_1 \xrightarrow{c_1} a_2$ or $a_0 \xrightarrow{b_2} a_2$ is a sub-trace of π;
- either b_1 and c_0, or b_2 are reached before a_2 in π.

Local Causality Graph

- Causality of a_2.
- Initial context $\varsigma = \{a \mapsto \{0\}; b \mapsto \{0\}; c \mapsto \{0, 1\}; d \mapsto \{0\}\}$.



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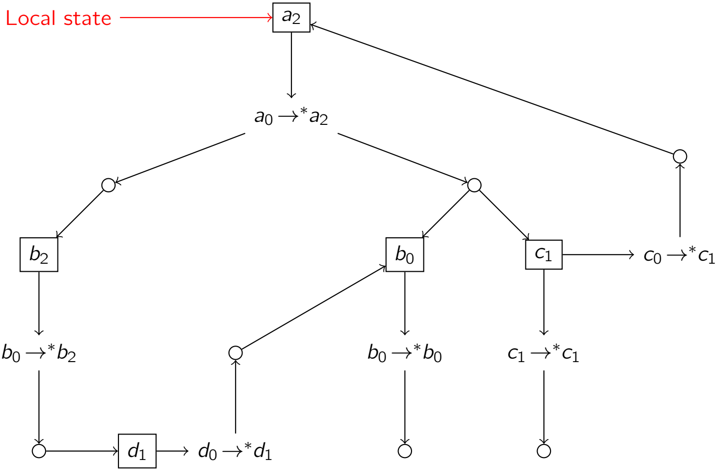

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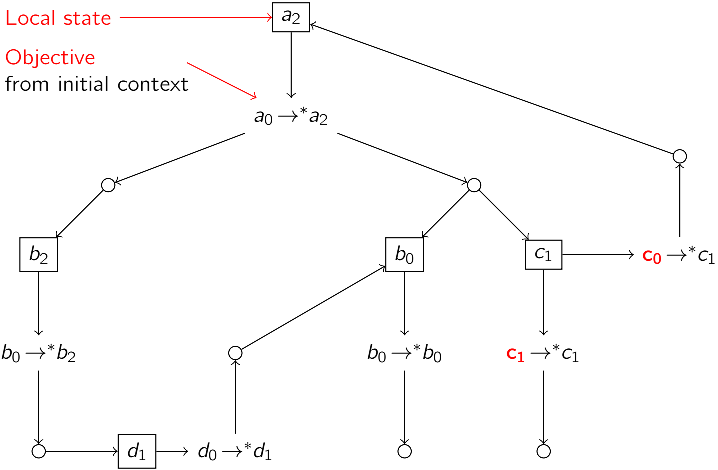

Local Causality Graph

- Causality of a_2.
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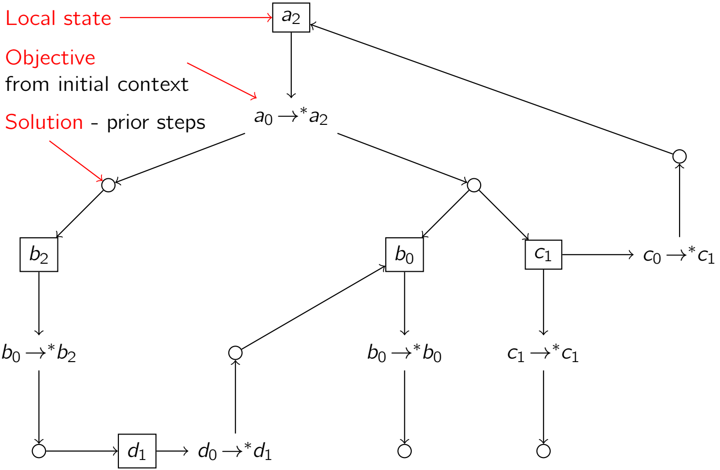

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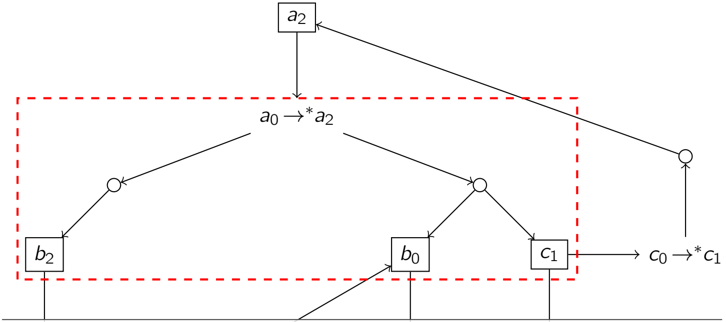


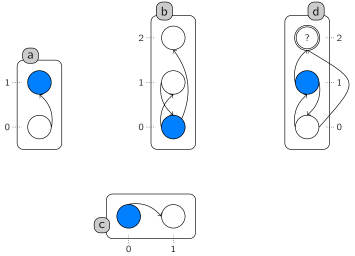
Objective completeness criteria

Objective is impossible from any state if at least one local state of each solution is disabled.

E.g. $a_0 \rightarrow^* a_2$ is impossible in $\mathcal{M} \ominus \{b_2, b_0\}$ and in $\mathcal{M} \ominus \{b_2, c_1\}$

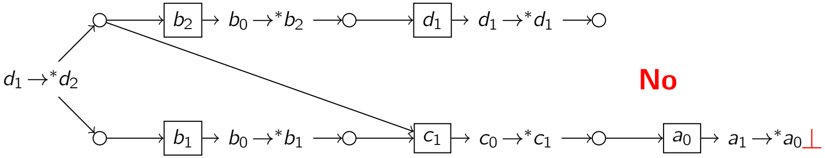
Necessary conditions for reachability

Example



Necessary condition for d_2 reachability from ς:
There exists a traversal of the LCG s.t.:

- objective $\rightarrow$ follow at least one solution;
- local state $\rightarrow$ follow all objectives;
- no cycle.



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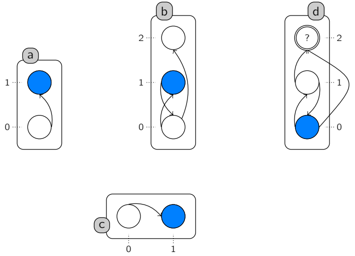

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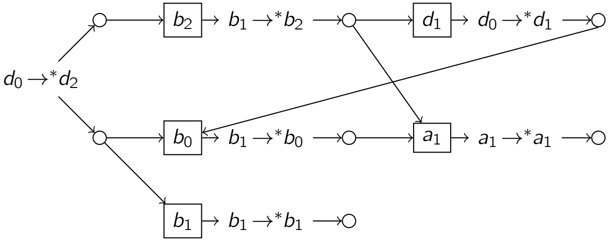

Inconc

Outline

Goal-oriented reduction

Motivations

Local causality analysis in previous work. . .

- . . . decide efficiently reachability properties (and cut sets)
- . . . but can be inconclusive (abstractions).

We may still want to do **exhaustive explorations** of the state space. . .

- . . . to ensure conclusiveness
- . . . to ensure that we are not missing cut-sets for reachability
- . . . to do any more precise analysis.

Can local causality analysis drives exhaustive analysis of the state space?

Model reduction

Model reductions

- merge/remove components/transitions,
- try to preserve some properties.

Goal-oriented reduction of automata networks

- dedicated to a given reachability property (reach a_i, then b_j, ...);
- reduction by removing transitions;
- conserve all minimal traces satisfying a reachability property.

Minimal traces (sequences of transitions)
A trace $\pi \models P$ is minimal w.r.t. P iff there is no sub-trace $\pi' \subsetneq \pi$ s.t. $\pi' \models P$.

Examples for $P =$ reach a_i:

- $b_0 \xrightarrow{c_0} b_1, c_0 \xrightarrow{b_1} c_1, a_0 \xrightarrow{b_1,c_1} a_i$ (YES)
- $b_0 \xrightarrow{c_0} b_1, c_0 \xrightarrow{b_1} c_1, d_0 \xrightarrow{c_1} d_1, a_0 \xrightarrow{b_1,c_1} a_i$ (NO)
- $b_0 \xrightarrow{c_0} b_1, c_0 \xrightarrow{b_1} c_1, b_1 \xrightarrow{a_0} b_0, d_0 \xrightarrow{c_1} d_1, b_0 \xrightarrow{d_1} b_1, a_0 \xrightarrow{b_1,c_1} a_i$ (NO)

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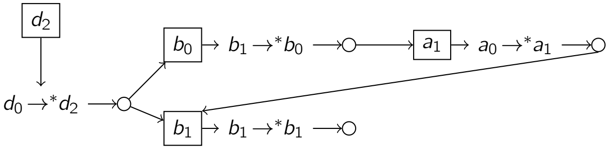
Reduction for single local reachability

Sketch

1. Compute LCG $\mathcal{G}$ from initial context for given local reachability property
2. Remove impossible objectives
3. Extends its context with local states nodes + intermediates given by local-cause
4. Repeat until fixpoint $\to \lceil\mathcal{G}\rceil$

$\Rightarrow$ keep only transitions in $\bigcup\{\text{tr}(\text{local-cause}(a_i \to^* a_j)) \mid a_i \to^* a_j \in \lceil\mathcal{G}\rceil\}$



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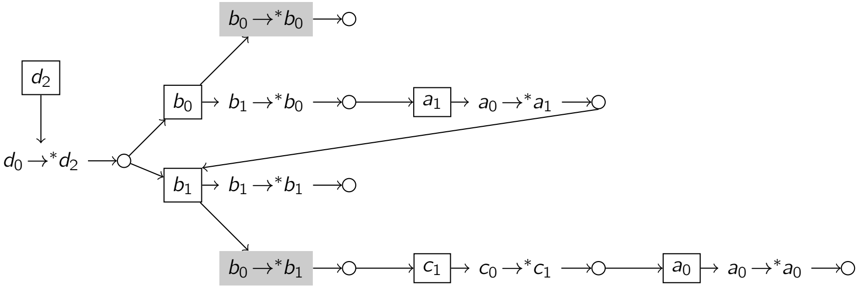

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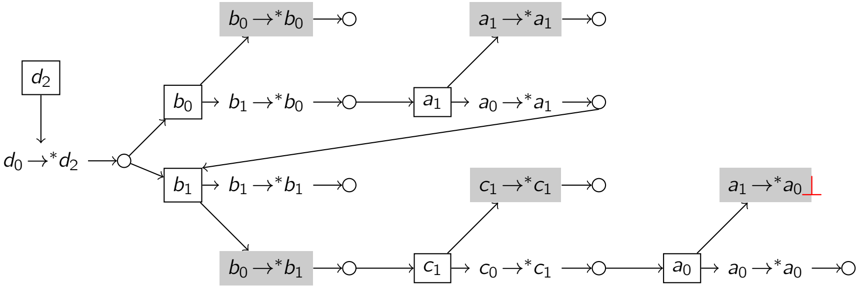


Goal-oriented reduction

Theorem

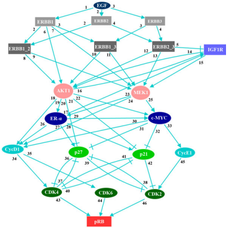
Given an AN $\mathcal{A} = (\Sigma, S, T)$, a global state $s \in S$, and one automaton local state a_i, for all minimal trace π from s to a_i, $\text{tr}(\pi) \subset \text{tr}(\lceil \mathcal{G} \rceil)$.

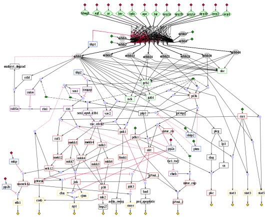
Consequence

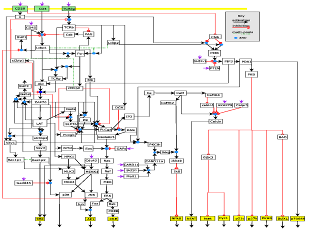
The AN $\mathcal{A} = (\Sigma, S, \text{tr}(\lceil \mathcal{G} \rceil))$ conserves all minimal traces for reaching a_i from s.

Results

Preliminary benchmarks with single reachability







Model		# tr	NuSMV time	NuSMV mem	ITS time	ITS mem	# states
Egf-r (20)	normal	68	0.1s	15Mb	0.35s	19Mb	4.200
	reduced	43	0.03s	11Mb	0.13s	8Mb	722
Egf-r (104) profile 1	normal	378	75s	2.1Gb	0.8s	750Mb	$\approx 10^7$
	reduced	0	-	-	-	-	1
Egf-r (104) profile 2	normal	378	KO	KO	540s	1.5Gb	$> 8.10^{14}$
	reduced	211	52s	100Mb	3.4s	100Mb	$\approx 6.10^7$
TCell-r (94)	normal	217	KO	KO	KO	KO	?
	reduced	42	10s	190Mb	0.25s	15Mb	60.000

For all cases, reduction step took between 0.01 and 0.1s.

Conclusion

Local Causality Graph (LCG)

- Abstract representation of the traces of ANs.
- Compact: poly(nb. automata), exp(size of 1 automaton).
- Exploits concurrency and causality.

Static analysis of transient reachability using LCG

- Necessary conditions for reachability in ANs.
- Sufficient conditions for reachability in ANs.
- Under-approximation of cut sets for reachability in ANs.

Goal-oriented reduction

- Intertwining between static and dynamics analysis
- Drives the exploration of the state space
- Applies to any update schedule

Research Direction

More properties from the Local Causality Graph

- Delimit complex attractors.
- Time scales (priorities between transitions).
- Cut sets that conserve a given property.

Goal-oriented reduction

- Reduction on the fly
- Mix with other static reductions (components)
- Embed in Petri net unfoldings

Applications in systems biology

- Drive model identification from time-series data
- Prediction for cell re-differenciation

SASB'15

6th International Workshop on Static Analysis and Systems Biology

8 September 2015 - Saint-Malo (France)

`https://www.lri.fr/sasb2015/`

Scope:

- Quantitative and qualitative models
- Topology vs dynamics
- Model reduction
- Abstract interpreration frameworks
- Practical methods for tackling biological models..

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Deadline for paper submission: 29th May 2015 (proceedings in ENTCS)

Questions?